Parameter Dependence of Pair Correlations In Clean Proximity Systems

Alberto Garcia - California State University, Long Beach

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Committee Members:
Andreas Bill, Dr. rer. nat. (Chair)
Michael Peterson, Ph.D.
Claudia Ojeda-Aristizabal, Ph.D.


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Outline:

- Background
  - Motivation
- Method
  - Theory
- Results
  - Discrete layers
  - Continuous layers
  - Ballistic/Diffusive Comparison
- Conclusion
  - Future work

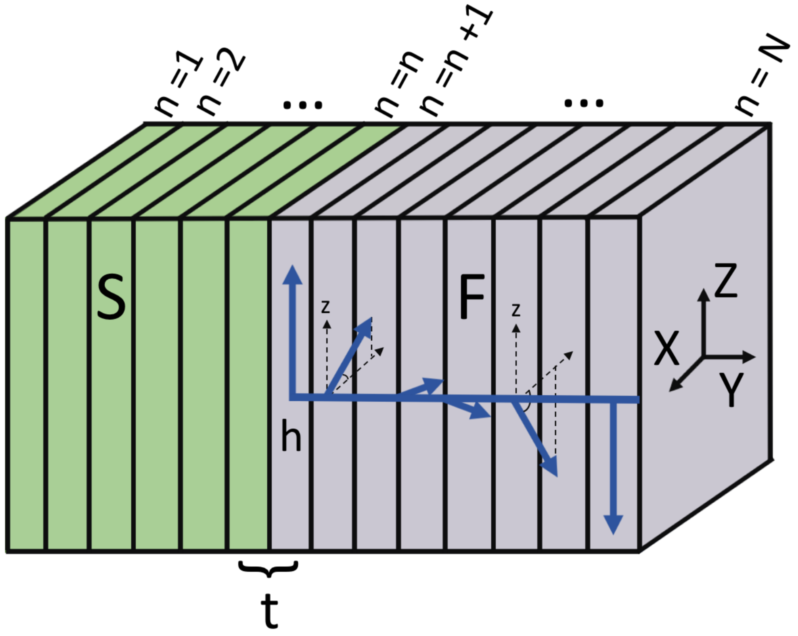
Background:

- Motivation
  - I. Focus of research
  - II. Applications

Focus of research

Question:

- How do varying magnetic configurations affect pair correlations within a Josephson junction?



Example: SF proximity system

Quasi one-dimensional system

Applications:

- Quantum computing
- Spintronics devices
- Memory storage
- Sensors

We will be looking at different magnetic systems

Method:

- Theory
  1. What are pair correlations?
  2. Hamiltonian
  3. Bogoliubov-de Gennes (BdG) equations
  4. Gor’kov functions

What are pair correlations?

Singlet

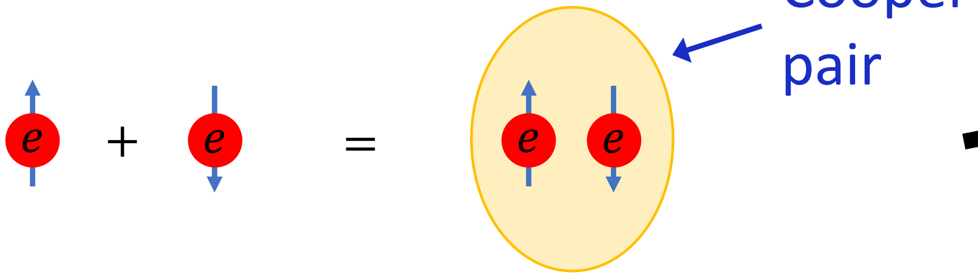
Triplet

$$\underbrace{|0,0\rangle \propto |\uparrow\downarrow - \downarrow\uparrow\rangle, \quad |1,0\rangle \propto |\uparrow\downarrow + \downarrow\uparrow\rangle,}_{\text{“Fast” decay}} \quad \underbrace{|1,1\rangle = |\uparrow\uparrow\rangle, \quad |1,-1\rangle = |\downarrow\downarrow\rangle.}_{\text{“Slow” decay}}$$

Pair potential:

$$\Delta_{\sigma,\sigma'} = \begin{pmatrix} \Delta_{\uparrow,\uparrow} & \Delta_{\uparrow,\downarrow} \\ \Delta_{\downarrow,\uparrow} & \Delta_{\downarrow,\downarrow} \end{pmatrix}$$

$$\Delta(n) = \frac{g}{2}\langle c_{n,\downarrow} c_{n,\uparrow} \rangle$$



- Two fermions (spin 1/2)

$$|\Psi\rangle_{spin} = \sum_{s=0,1} \sum_{m=0,\pm 1} \alpha_{s,m} \mid s,m >$$

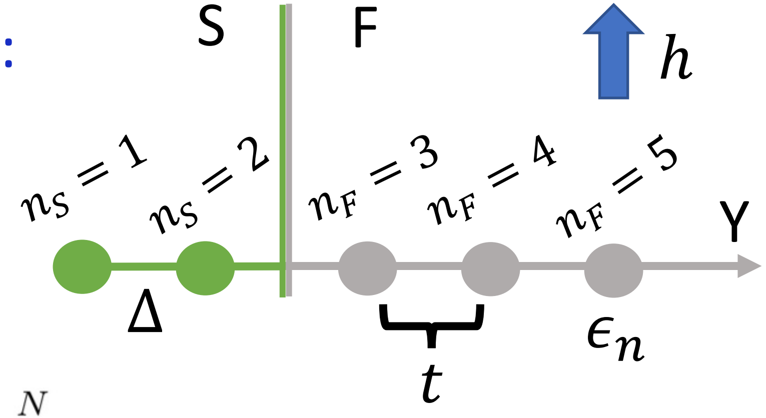
$$s = 0,1,$$
$$m = 0, \pm 1$$

Pairs are bound in the superconductor and leak into the magnetic material

Hamiltonian

- Work is done in the Clean limit
- Formalism (Bogoliubov – de Gennes)

Ex:



Non-zero in superconducting layers

Non-zero in magnetic layers

$$\mathcal{H} = -t\sum_{n}^{N}\sum_{\sigma}(c^{\dagger}_{n,\sigma}c_{n+1,\sigma} + h.c.) + \sum_{n}^{N}\sum_{\sigma}(\epsilon_n - \mu)c^{\dagger}_{n,\sigma}c_{n,\sigma}$$
$$+\sum_{n}^{N}\sum_{\sigma,\sigma'}\left(\Delta(n)\rho_{\sigma,\sigma'}c^{\dagger}_{n,\sigma}c^{\dagger}_{n,\sigma'} + h.c.\right) - \sum_{n}^{N}\sum_{\sigma,\sigma'}[\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{h}(n)]_{\sigma,\sigma'}c^{\dagger}_{n,\sigma}c_{n,\sigma'}$$

- t is the nearest neighbor hopping energy,
- ϵ_n is the local energy on site *n*, and μ the chemical potential,
- $\Delta(n)$ is the pair potential (depends on *n*),
- $h(n)$ is the magnetization profile in the magnetic material,
- $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices.

Solve the associated Bogoliubov – de Gennes equations for the tight-binding model.

Bogoliubov-Valatin Transformation

- The Bogoliubov-Valatin (BV) Transformation is used to diagonalize the Hamiltonian.

Creation-annihilation operators:

$$c_{n,\sigma} = \sum_{J}^{M} \left(u_{n,\sigma,J}\gamma_J - \mathcal{S}_\sigma v^*_{n,\sigma,J}\gamma^\dagger_J \right)$$

$$c^\dagger_{n,\sigma} = \sum_{J}^{M} \left(u^*_{n,\sigma,J}\gamma^\dagger_J - \mathcal{S}_\sigma v_{n,\sigma,J}\gamma_J \right)$$

$$\mathcal{S}_\sigma = \begin{cases} 1 & \text{if } \sigma = \uparrow \\ -1 & \text{if } \sigma = \downarrow \end{cases}$$

→

Commutation relations:

$$\left[c_{n,\sigma}, c_{n',\sigma'}\right]_+ = \left[c^\dagger_{n,\sigma}, c^\dagger_{n',\sigma'}\right]_+ = 0,$$

$$\left[c_{n,\sigma}, c^\dagger_{n',\sigma'}\right]_+ = \delta_{n,n'}\delta_{\sigma,\sigma'}.$$

Condition for diagonalizing Hamiltonian:

$$[\mathcal{H}, \gamma_J]_- = -E_j\gamma_J$$

$$\left[\mathcal{H}, \gamma^\dagger_J\right]_- = E_j\gamma^\dagger_J$$

Bogoliubov-de Gennes equations

$$\begin{aligned}
u_{n,\uparrow,J}E_J &= -t(u_{n+1,\uparrow,J}+u_{n-1,\uparrow,J})+[\epsilon_n-\mu-h_z(n)]u_{n,\uparrow,J}\\
&\quad +\Delta(n)v_{n,\downarrow,J}-[h_x(n)-ih_y(n)]u_{n,\downarrow,J}\\
u_{n,\downarrow,J}E_J &= -t(u_{n+1,\downarrow,J}+u_{n-1,\downarrow,J})+[\epsilon_n-\mu+h_z(n)]u_{n,\downarrow,J}\\
&\quad +\Delta(n)v_{n,\uparrow,J}-[h_x(n)+ih_y(n)]u_{n,\uparrow,J}\\
v_{n,\uparrow,J}E_J &= t(v_{n+1,\uparrow,J}+v_{n-1,\uparrow,J})-[\epsilon_n-\mu-h_z(n)]v_{n,\uparrow,J}\\
&\quad +\Delta^*(n)u_{n,\downarrow,J}-[h_x(n)+ih_y(n)]v_{n,\downarrow,J}\\
v_{n,\downarrow,J}E_J &= t(v_{n+1,\downarrow,J}+v_{n-1,\downarrow,J})-[\epsilon_n-\mu+h_z(n)]v_{n,\downarrow,J}\\
&\quad +\Delta^*(n)u_{n,\uparrow,J}-[h_x(n)-ih_y(n)]v_{n,\uparrow,J}.
\end{aligned}$$

$$\begin{pmatrix} H & -\hat{M} & 0 & \hat{\Delta} \\ -\hat{M}^* & H' & \hat{\Delta} & 0 \\ 0 & \hat{\Delta}^* & -H & -\hat{M}^* \\ \hat{\Delta}^* & 0 & -\hat{M} & -H' \end{pmatrix} \begin{pmatrix} U_{\uparrow,J} \\ U_{\downarrow,J} \\ V_{\uparrow,J} \\ V_{\downarrow,J} \end{pmatrix} = E_j \begin{pmatrix} U_{\uparrow,J} \\ U_{\downarrow,J} \\ V_{\uparrow,J} \\ V_{\downarrow,J} \end{pmatrix}$$

$$U_{\sigma J} = \begin{pmatrix} u_{1,\sigma,J} \\ u_{2,\sigma,J} \\ \vdots \\ u_{n,\sigma,J} \end{pmatrix}, \qquad V_{\sigma J} = \begin{pmatrix} v_{1,\sigma,J} \\ v_{2,\sigma,J} \\ \vdots \\ v_{n,\sigma,J} \end{pmatrix}$$

$u_{n,\sigma}$ and $v_{n,\sigma}$ are
particle-hole amplitudes

Block matrix for H:

$$H = \begin{pmatrix} \epsilon_1 - \mu - h_z(1) & -t & 0 & \cdots & 0 \\ -t & \epsilon_2 - \mu - h_z(2) & -t & \ddots & \vdots \\ 0 & -t & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -t \\ 0 & \cdots & 0 & -t & \epsilon_n - \mu - h_z(N) \end{pmatrix}$$

Block matrix for $\hat{\Delta}$:

$$\hat{\Delta} = \begin{pmatrix} \Delta(1) & 0 & \cdots & 0 \\ 0 & \Delta(2) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Delta(N) \end{pmatrix}$$

Block matrix for $\widehat{M}$:

$$\hat{M} = \begin{pmatrix} h_x(1) - ih_y(1) & 0 & \cdots & 0 \\ 0 & h_x(2) - ih_y(2) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_x(N) - ih_y(N) \end{pmatrix}$$

Physical quantities

- Using the BdG solution, other quantities can be defined in the particle-hole basis.

➢ Pair potential: $$\Delta(n) = \frac{g}{2}\sum_{J}^{M}(u_{n,\uparrow,J}v^{*}_{n,\downarrow,J} + u_{n,\downarrow,J}v^{*}_{n,\uparrow,J})[1-2f(E_j)]$$

Fractional filling

➢ Number of particles: $$N = \sum_{n,\sigma}\sum_{J}^{M}\left\{|u_{n,\sigma,J}|^2 f(E_j) + |v_{n,\sigma,J}|^2[1-f(E_j)]\right\} \rightarrow \quad n = \frac{N}{N_{tot}}$$

➢ Local density of states: $$\rho(n,E) = \lim_{\eta\to 0}\frac{\eta}{2\pi}\sum_{n}^{N}\sum_{J}^{M}\left[\frac{|u_{n,\uparrow,J}|^2 + |u_{n,\downarrow,J}|^2}{(E-E_j)^2 + (\eta/2)^2} + \frac{|v_{n,\uparrow,J}|^2 + |v_{n,\downarrow,J}|^2}{(E+E_j)^2 + (\eta/2)^2}\right]$$

➢ Gor’kov functions: $$f_{\sigma,\sigma'}(n,\tau) = \frac{1}{2}\langle c_{n,\sigma}(\tau)c_{n,\sigma'}(0)\rangle \qquad \sigma,\sigma' \in \{\uparrow,\downarrow\}$$

➢ Josephson current

Gor’kov functions

$s = 0, 1,$

$m = 0, \pm 1$

- The pair correlations can be described by the Gor’kov functions.
- General definition:

$$f_{\sigma,\sigma'}(n,\tau) = \frac{1}{2}\langle c_{n,\sigma}(\tau) c_{n,\sigma'}(0)\rangle \qquad \sigma, \sigma' \in \{\uparrow, \downarrow\}$$

Singlet:

$$f_{0,0} = f_{\uparrow,\downarrow} - f_{\downarrow,\uparrow} = \frac{1}{2}\left\langle c_{n,\uparrow} c_{n,\downarrow} - c_{n,\downarrow} c_{n,\uparrow}\right\rangle$$

$$f_{0,0}(n) = \frac{1}{2}\sum_{J}^{M}(u_{n,\uparrow,J} v^*_{n,\downarrow,J} + u_{n,\downarrow,J} v^*_{n,\uparrow,J})\left[1 - 2f(E_j)\right]$$

Triplets:

$$f_{1,0} = \frac{1}{2}\left(f_{\uparrow,\downarrow} + f_{\downarrow,\uparrow}\right)$$

$$f_{1,1} = \frac{1}{2}\left(f_{\uparrow,\uparrow} - f_{\downarrow,\downarrow}\right)$$

$$f_{1,0}(n) = \frac{1}{2}\sum_{J}^{M}\left[u_{n,\uparrow,J} v^*_{n,\downarrow,J} - u_{n,\downarrow,J} v^*_{n,\uparrow,J}\right]\zeta_j(\tau),$$

$$f_{1,1}(n) = -\frac{1}{2}\sum_{J}^{M}\left[u_{n,\uparrow,J} v^*_{n,\uparrow,J} + u_{n,\downarrow,J} v^*_{n,\downarrow,J}\right]\zeta_j(\tau)$$

Time-dependence

- The triplet Gor'kov functions are time dependent.
- We apply a Fourier transform to transform the equations from the time domain to the frequency domain:

Triplets:

$$f_{1,0}(n) = \frac{1}{2}\sum_{J}^{M}\Big[u_{n,\uparrow,J}v^{*}_{n,\downarrow,J} - u_{n,\downarrow,J}v^{*}_{n,\uparrow,J}\Big]\zeta_j(\tau),$$

$$f_{1,1}(n) = -\frac{1}{2}\sum_{J}^{M}\Big[u_{n,\uparrow,J}v^{*}_{n,\uparrow,J} + u_{n,\downarrow,J}v^{*}_{n,\downarrow,J}\Big]\zeta_j(\tau)$$

Time-dependent operators:

$$c_{n,\sigma}(t) = e^{\frac{i\mathcal{H}t}{\hbar}}c_{n,\sigma}e^{-\frac{i\mathcal{H}t}{\hbar}};$$

$$c^{\dagger}_{n,\sigma}(t) = e^{\frac{i\mathcal{H}t}{\hbar}}c^{\dagger}_{n,\sigma}e^{-\frac{i\mathcal{H}t}{\hbar}}.$$

Time-dependent term:

$$\zeta_j(\tau) = \cos\left(\frac{E_j}{\hbar}\tau\right) - i\sin\left(\frac{E_j}{\hbar}\tau\right)[1 - 2f(E_j)].$$

Fourier transform:

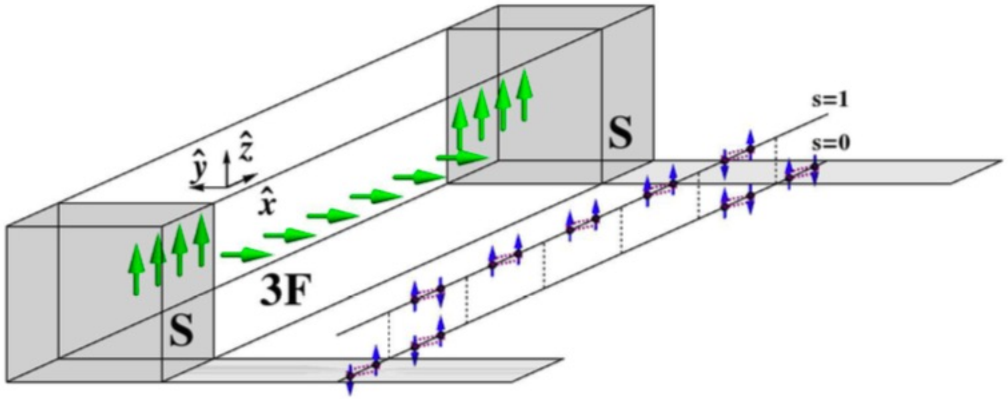
Frequency-dependent term:

$$\zeta_j(\omega) = \int_{-\infty}^{\infty}\zeta_j(\tau)e^{-i\omega\tau}d\tau, \rightarrow \zeta_j(\omega) = \pi[\delta(\omega - E_j) + \delta(\omega + E_j)] - \pi[1 - 2f(E_j)][\delta(\omega - E_j) - \delta(\omega + E_j)].$$

Discrete/Continuous configurations

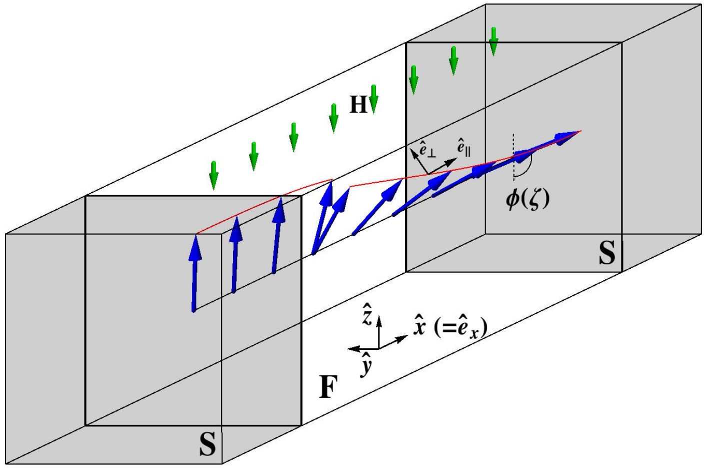
- Research done by group involves discrete and continuous layers.
- We compare the behavior of the pair correlations in both types of layers.

Discrete magnetization
(Ex: S3FS)



Ferromagnetic interface is located at specific sites

Continuous magnetization
(Ex: Helical)



Ferromagnetic interface is located at each of sites

Homogeneous vs. Inhomogeneous magnetization

$$\left|s,m\right\rangle = \left|0,0\right\rangle$$ Singlet superconductor

Homogeneous F ⟶ $\Downarrow$

$$\alpha_{0,0}\,|0,0> + \alpha_{1,0}\,|1,0>$$ (FFLO state)

$$+\cancel{\alpha_{1,1}\,|1,1> + \alpha_{1,-1}\,|1,-1>}$$

Only the singlet and $m = 0$ triplet are seen (fast decay)

Rotation of magnetization ⟶ $\Downarrow$

$$\sum_{s=0,1}\sum_{m=0,\pm 1}\alpha_{s,m}\,|s,m>$$

$$\left|0,0\right\rangle \quad \Rightarrow \quad \alpha_{0,0}\,|0,0> + \alpha_{1,0}\,|1,0> + \alpha_{1,1}\,|1,1> + \alpha_{1,-1}\,|1,-1>$$

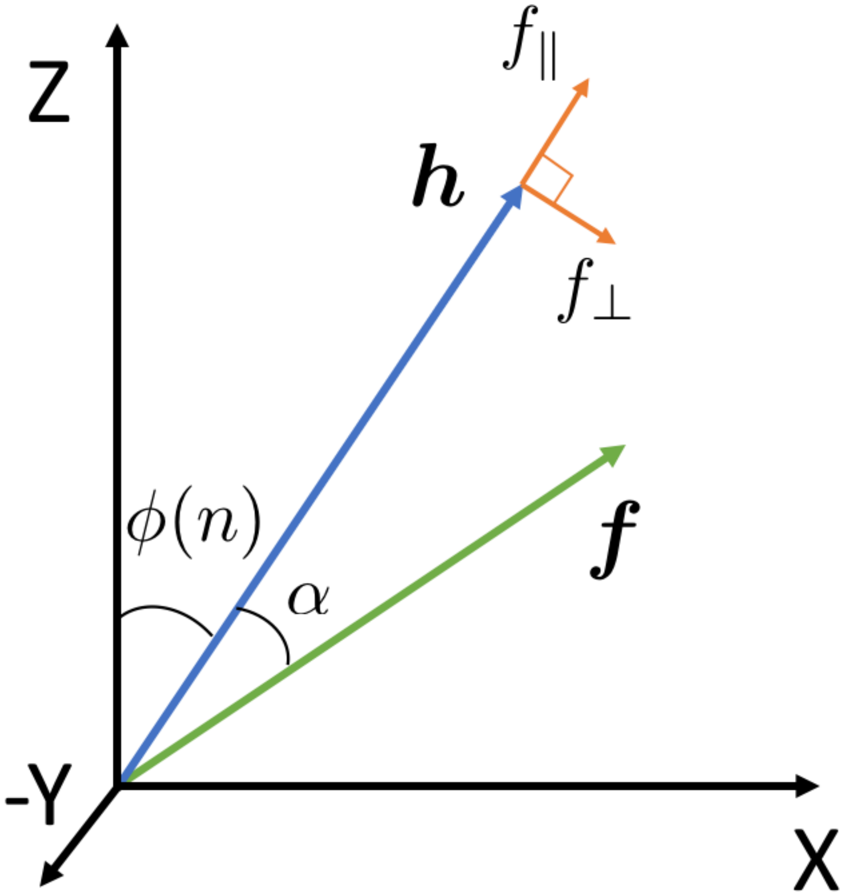
The $m = \pm 1$ triplets appear (slow decay) when the quantization axis is rotated (mixing)

✓ We study the mixing of $\left|s,m\right\rangle$ states at each rotation of the magnetization.

Rotation of quantization axis

Magnetization vector:

$$\boldsymbol{h}(n) = |\boldsymbol{h}|\sin\phi(n)\hat{\mathbf{x}} + |\boldsymbol{h}|\cos\phi(n)\hat{\mathbf{z}}$$



Gor’kov vector:

$$\boldsymbol{f}(y) = f_x(y)\hat{\mathbf{x}} + f_z(y)\hat{\mathbf{z}}.$$

Gor’kov vector with angle dependence:

$$\begin{aligned}\boldsymbol{f}(y) &= |\boldsymbol{f}|\left[\sin(\phi+\alpha), 0, -\cos(\phi+\alpha)\right]_{x,y,z}\\ &= |\boldsymbol{f}|\left(\cos\alpha, 0, -\sin\alpha\right)_{\perp,y,\|}\end{aligned}$$

Transformation:

$$\begin{pmatrix} f_{\perp} \\ f_{\|} \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} f_x \\ f_z \end{pmatrix} = \begin{pmatrix} f_x\cos\phi - f_z\sin\phi \\ f_x\sin\phi + f_z\cos\phi \end{pmatrix}$$

This rotation leads to →

$$f_{1,0} = f_{\|}$$

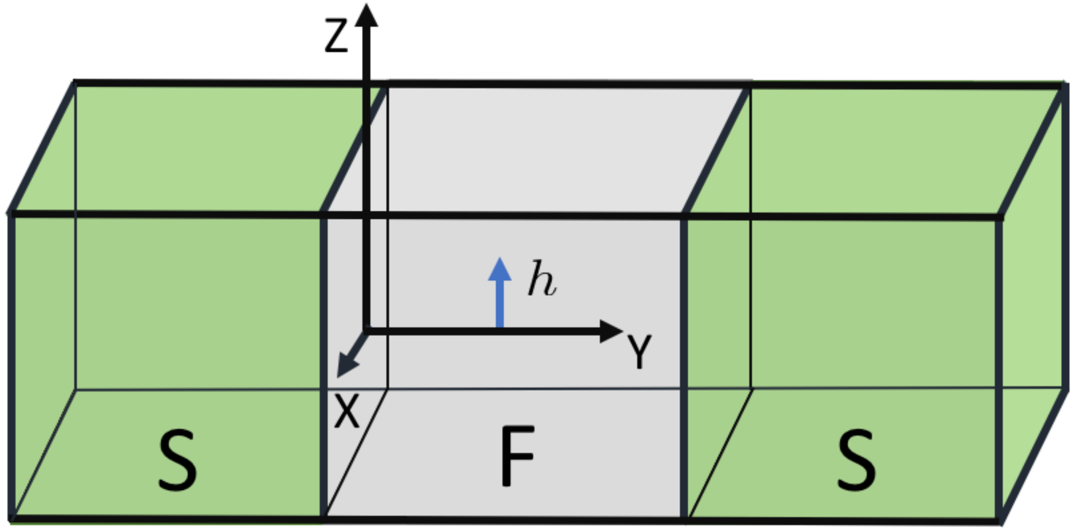
$$f_{1,1} = f_{\perp}$$

Results:

- ➢Discrete layers
  - I. Monolayer, Trilayer, Pentalayer
- ➢Continuous layers (Helical)
- ➢Comparing Ballistic and Diffusive regime

Discrete layers: Monolayer (SFS)

$$|0{,}0\rangle \implies \alpha_{0,0}|0{,}0\rangle + \alpha_{1,0}|1{,}0\rangle$$



- Composed of a superconductor-ferromagnet-superconductor.
- Homogeneous magnetization along the z-axis (up).

Functional form:

$$f_{s,0} \sim \frac{1}{y} e^{-\frac{y}{\xi_N}} \cos\left(\frac{y}{\xi_F} + s\frac{\pi}{2}\right)$$

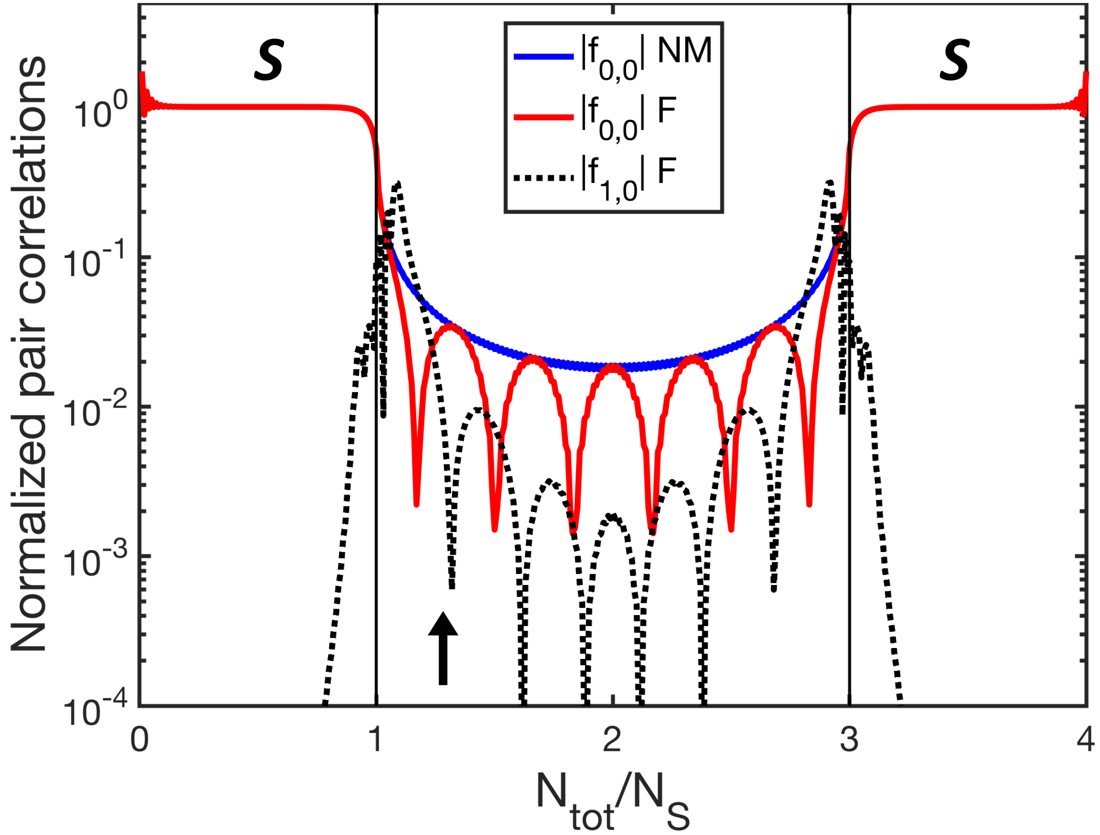
$$\xi_N = \frac{\hbar v_F}{2\pi T}, \quad \xi_F = \frac{\hbar v_F}{2\pi h}$$

Normalization of pair correlations:

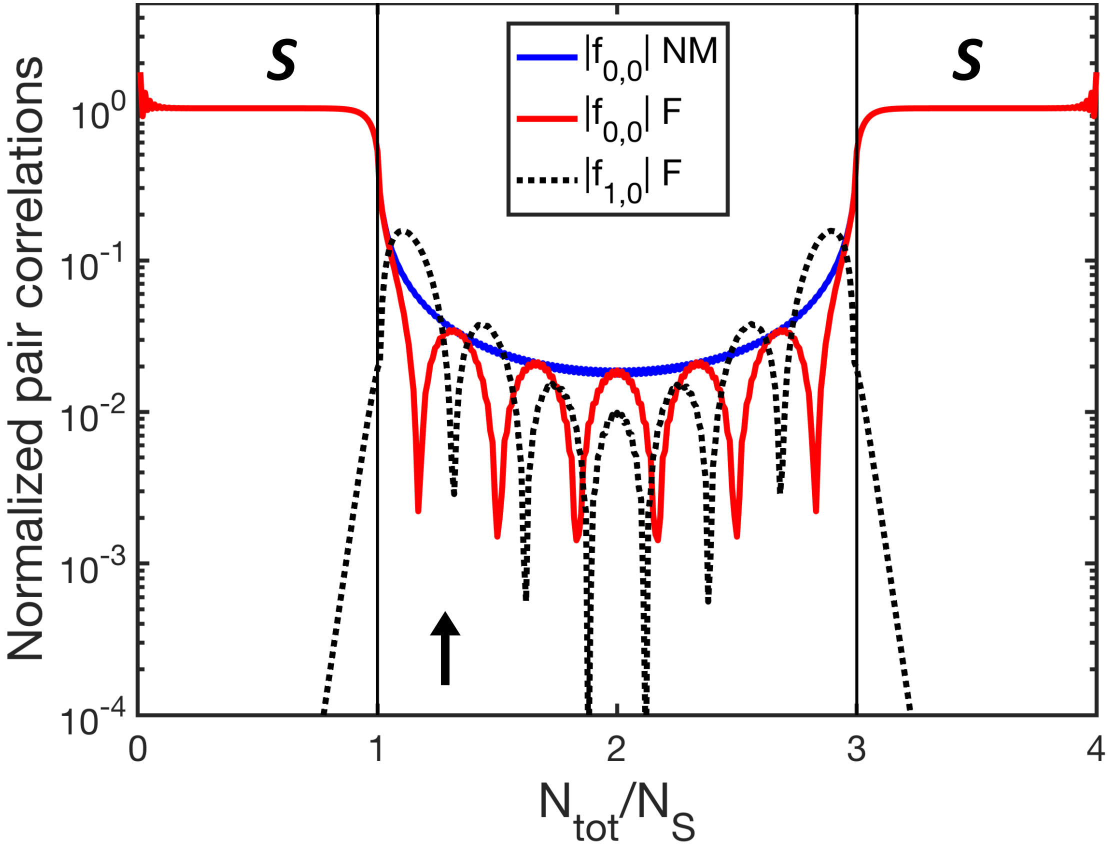
$$f_{s,m}^{norm} = \frac{f_{s,m}}{f_{0,0}^{avg}}$$

$n = 0.5$
$h = 0.1\mathrm{t}$

SFS Junction: Time vs. Frequency Domain



Pair correlations in the time domain: $\tau = 10$



Pair correlations in the frequency domain: $\omega = 0.1t$

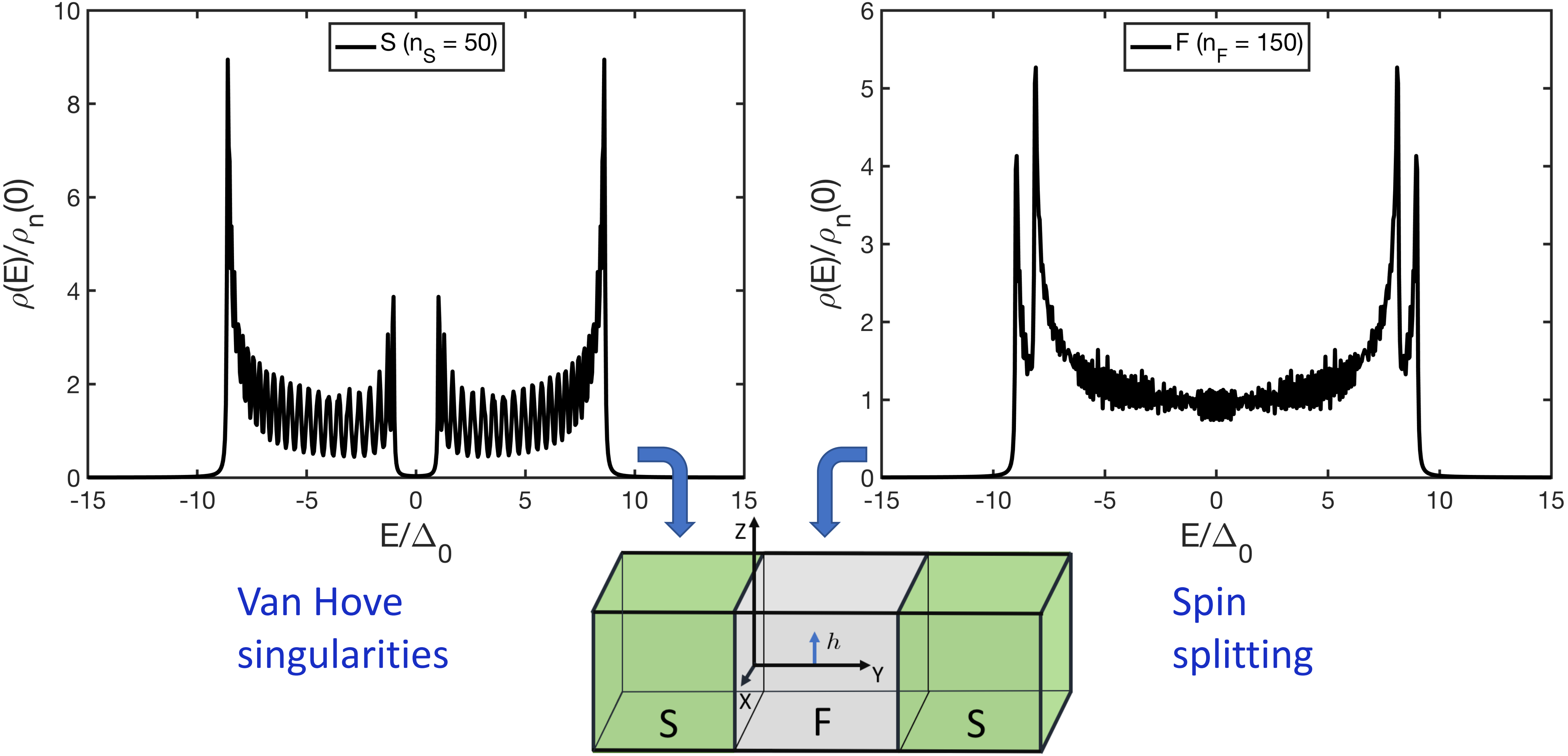
We will now only present pair correlations in the frequency regime n = 0.5

h = 0.1t

SFS Junction: Local Density of States (LDoS)

LDoS inside superconductor

LDoS inside ferromagnet

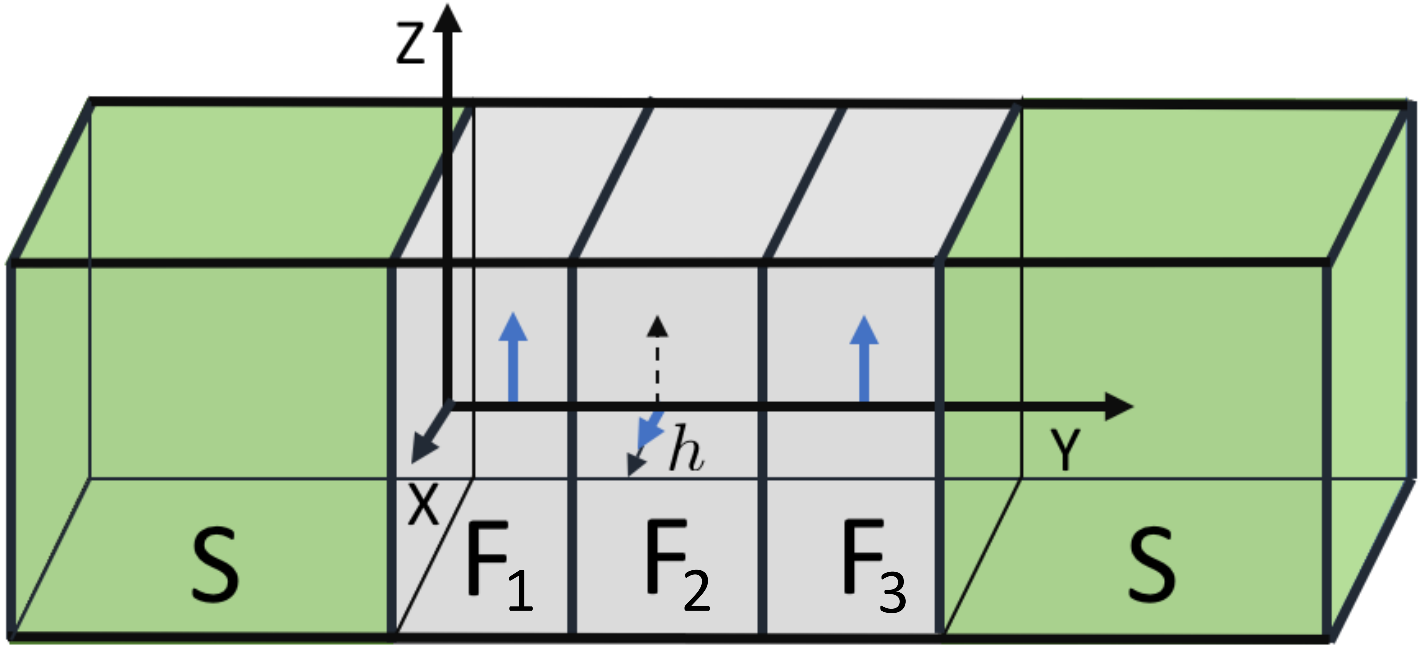


Van Hove singularities

Spin splitting

Discrete layers: Trilayer (S3FS)

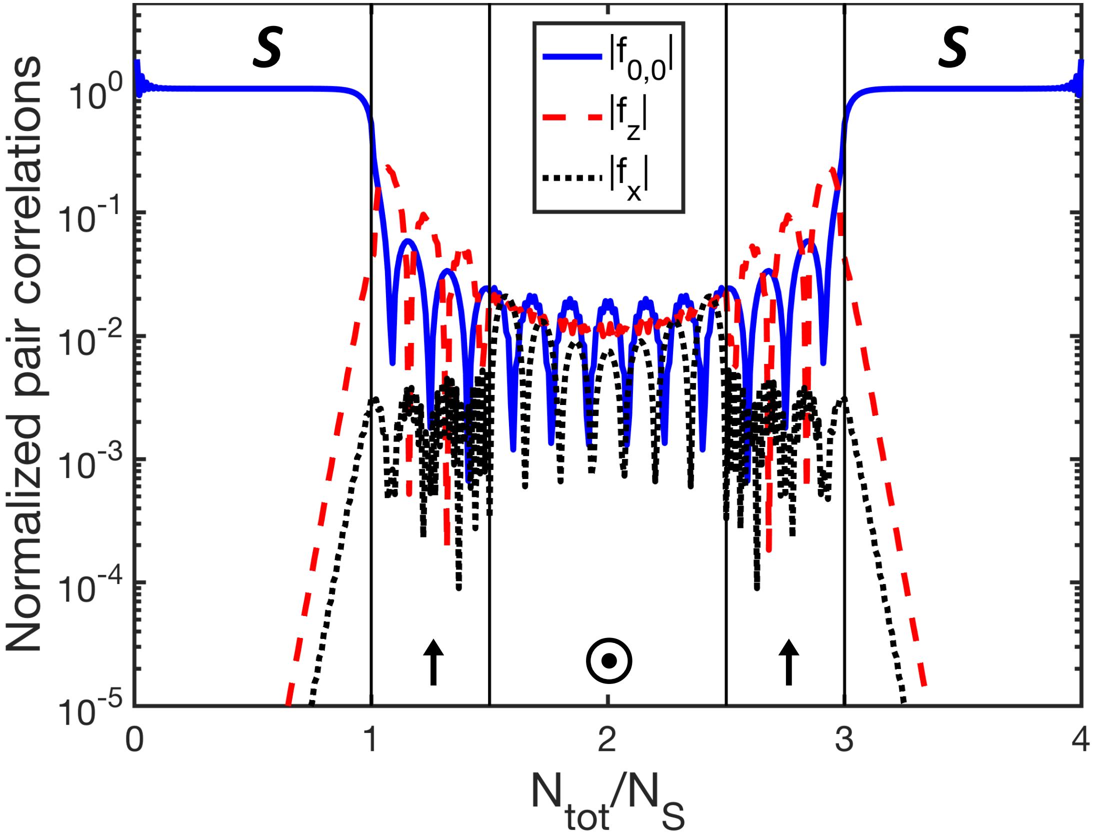
$$|0,0\rangle \implies \alpha_{0,0}|0,0\rangle + \alpha_{1,0}|1,0\rangle + \alpha_{1,1}|1,1\rangle + \alpha_{1,-1}|1,-1\rangle$$



- Josephson junction with magnetic material made up of three ferromagnets.
- Magnetization direction is up, out, up.

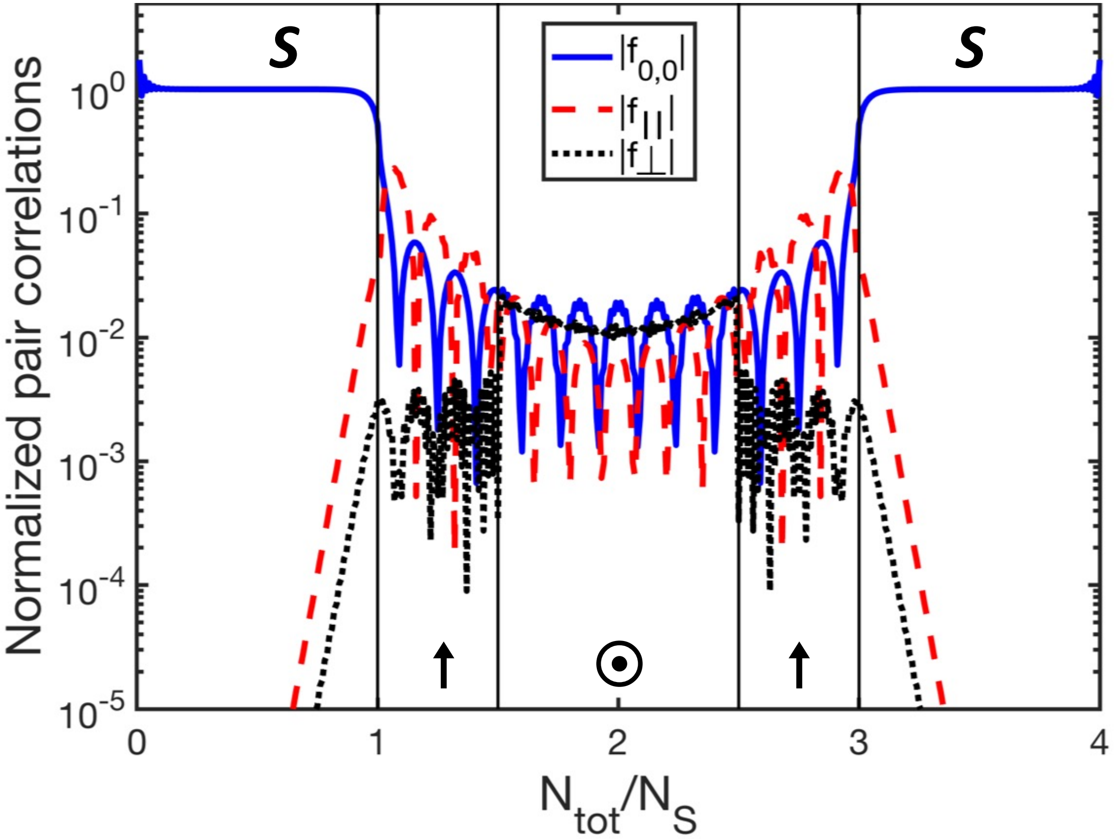
S3FS Junction: Cartesian vs. Rotating Basis

$n = 0.5$
$h = 0.2t$
$\omega = 0.1t$



Pair correlations in Cartesian (static) basis

$f_z \neq f_{1,0}$, $f_x \neq f_{1,1}$



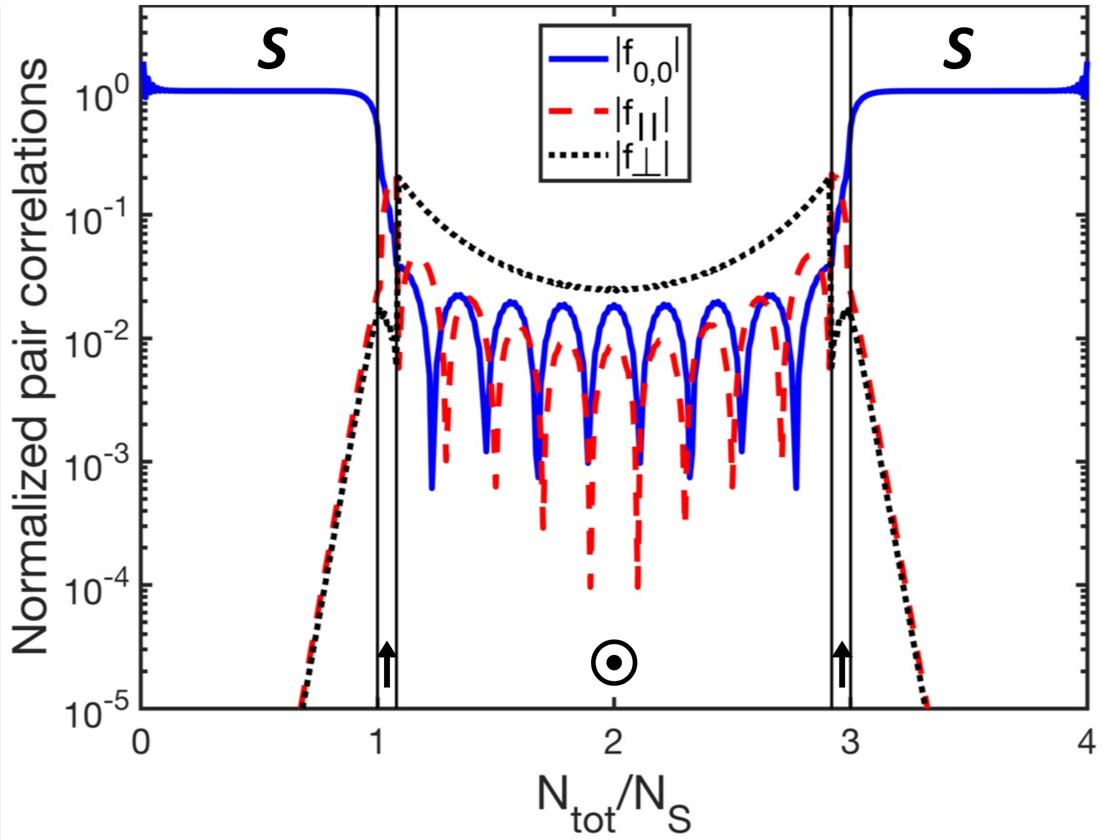
Pair correlations in rotating basis

$f_{\|} = f_{1,0}$, $f_{\perp} = f_{1,1}$

S3FS Junction: Width Dependence

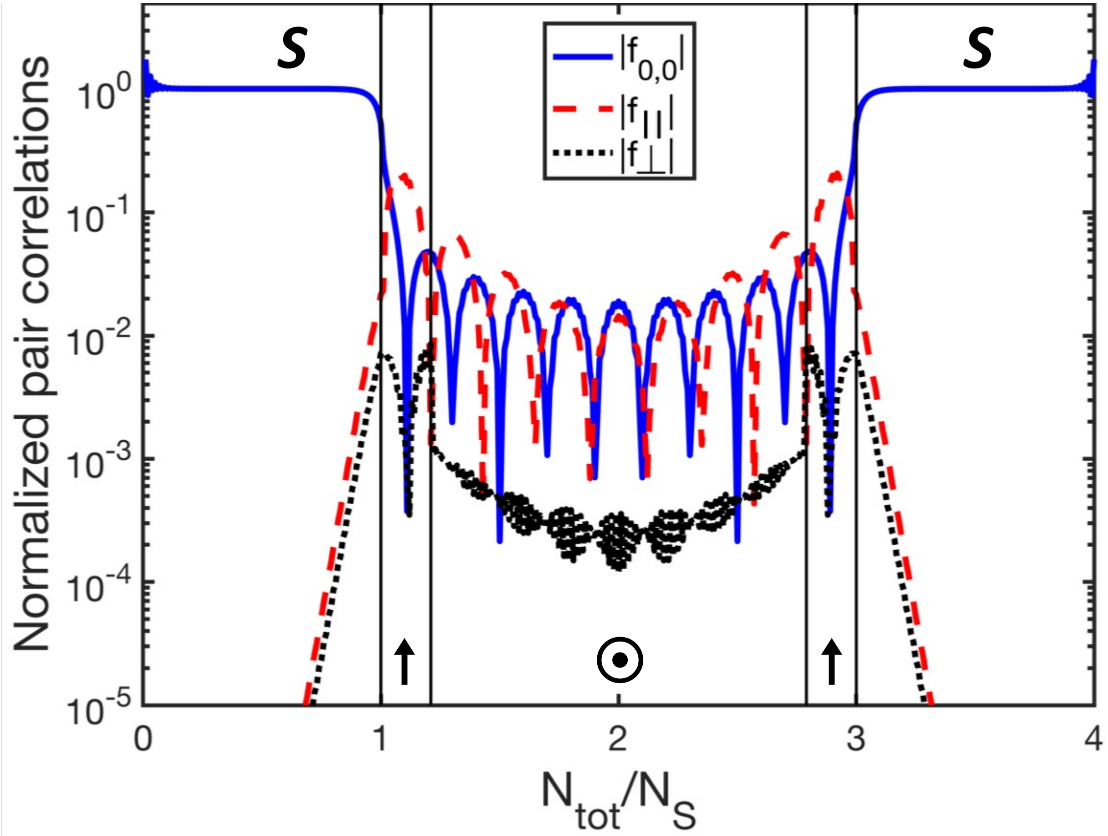
$n = 0.5$
$h = 0.15\mathrm{t}$
$\omega = 0.1t$

$|f_{\parallel}|$ maximum at interface



$N_{F_1} = N_{F_3} = 8, N_{F_2} = 184$

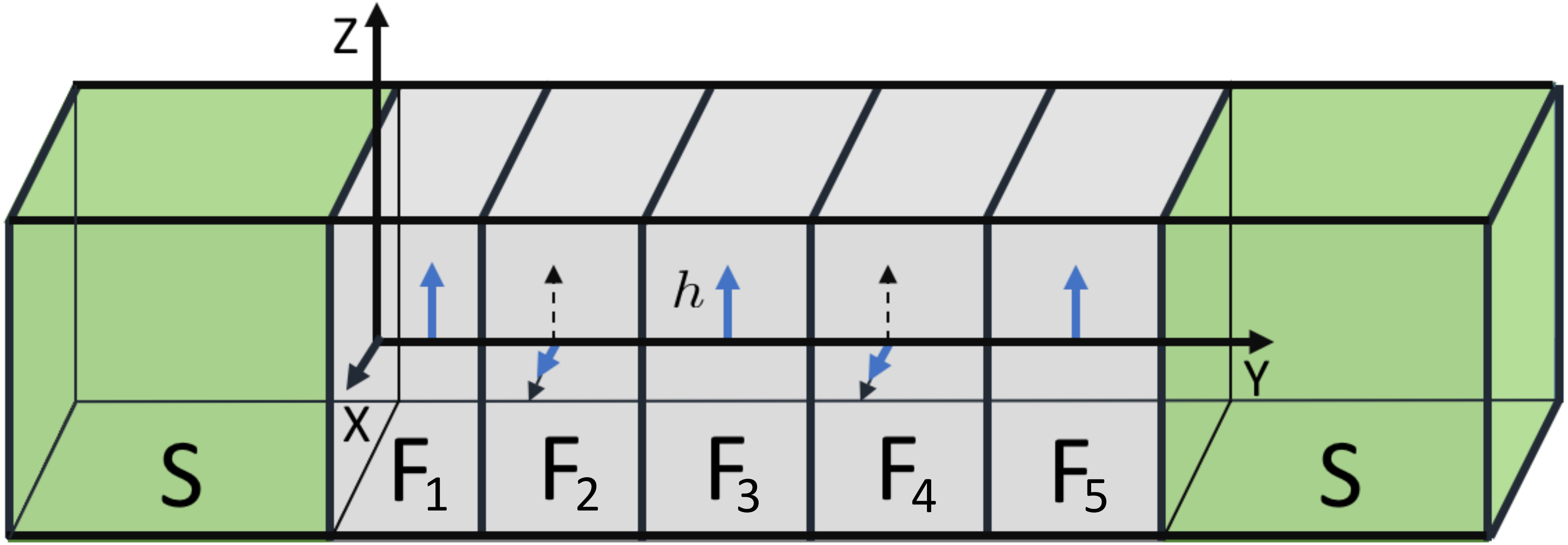
$|f_{\parallel}|$ minimum at interface



$N_{F_1} = N_{F_3} = 21, N_{F_2} = 158$

Discrete layers: Pentalayer (S5FS)

$$|0,0\rangle \implies \alpha_{0,0}|0,0\rangle + \alpha_{1,0}|1,0\rangle + \alpha_{1,1}|1,1\rangle + \alpha_{1,-1}|1,-1\rangle$$

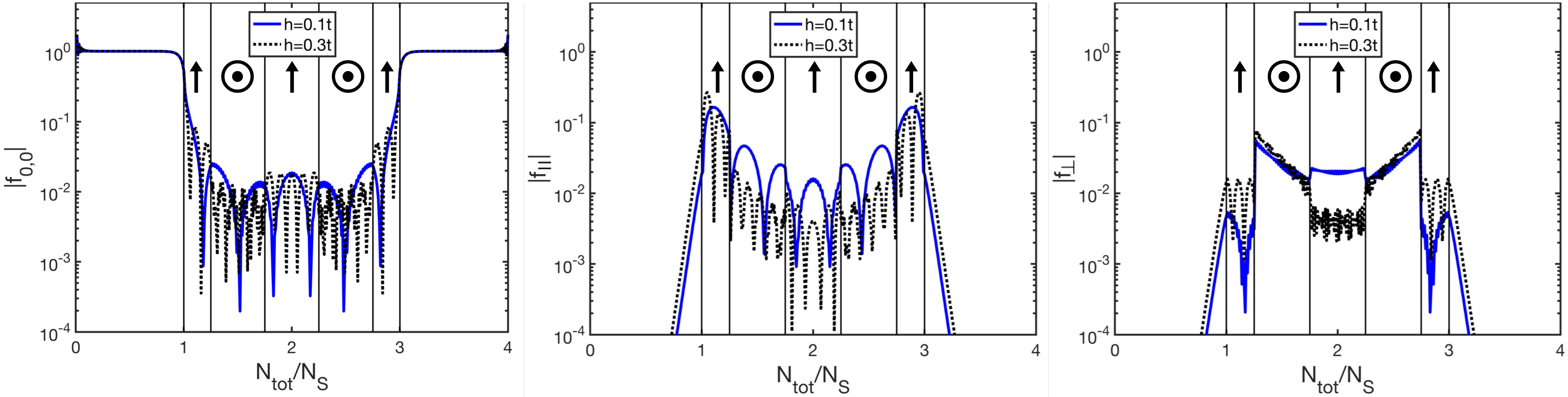


- Josephson junction with magnetic material composed of five ferromagnets.
- Magnetization direction is up, out, up, out, up.

$n = 0.5$
$\omega = 0.1t$

S5FS Junction: Magnetization Dependence

$|f_{0,0}|$ $|f_{\parallel}|$ $|f_{\perp}|$

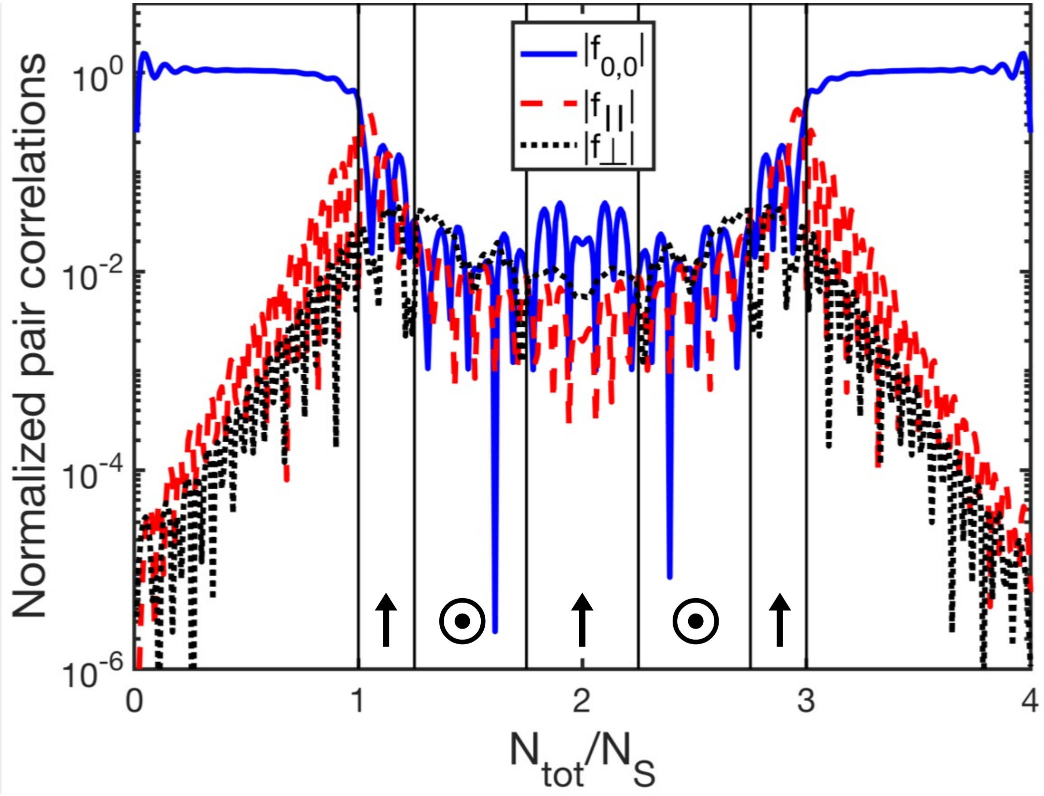


Blue smooth curve is $h = 0.1t$
Black dotted curve is $h = 0.3t$

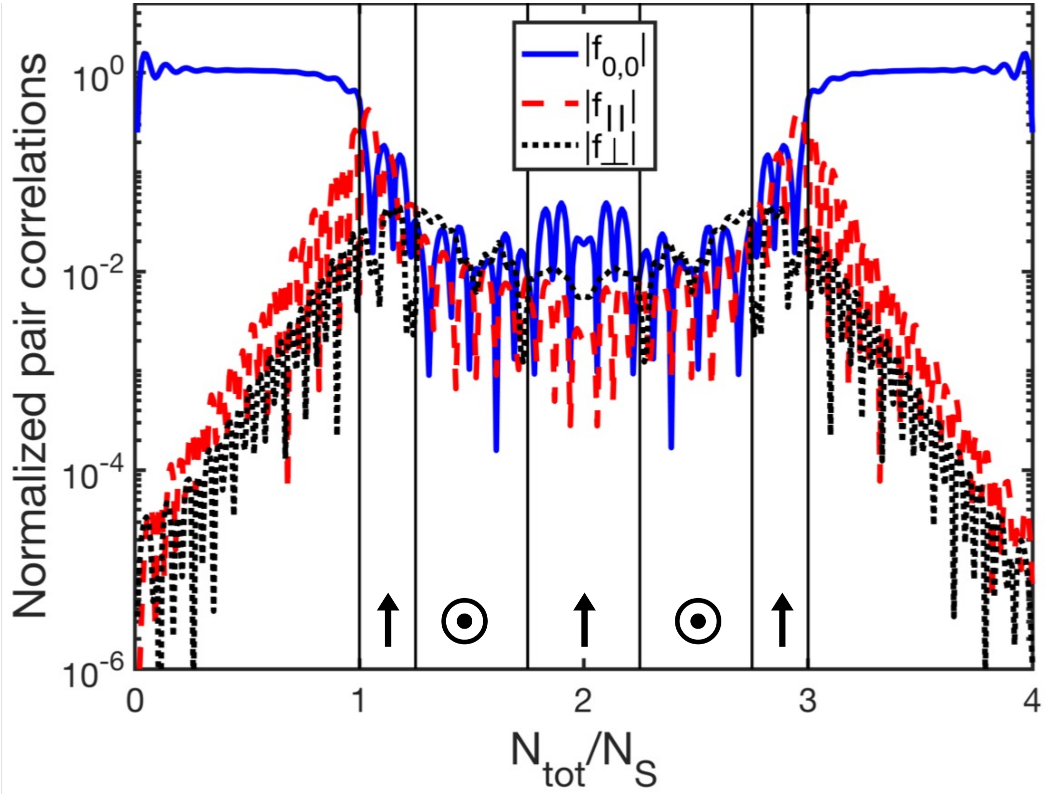
S5FS Junction: Particle-Hole Symmetry

$h = 0.1\mathrm{t}$
$\omega = 0.1t$

Filling: n = 0.1 (Nearly-empty)

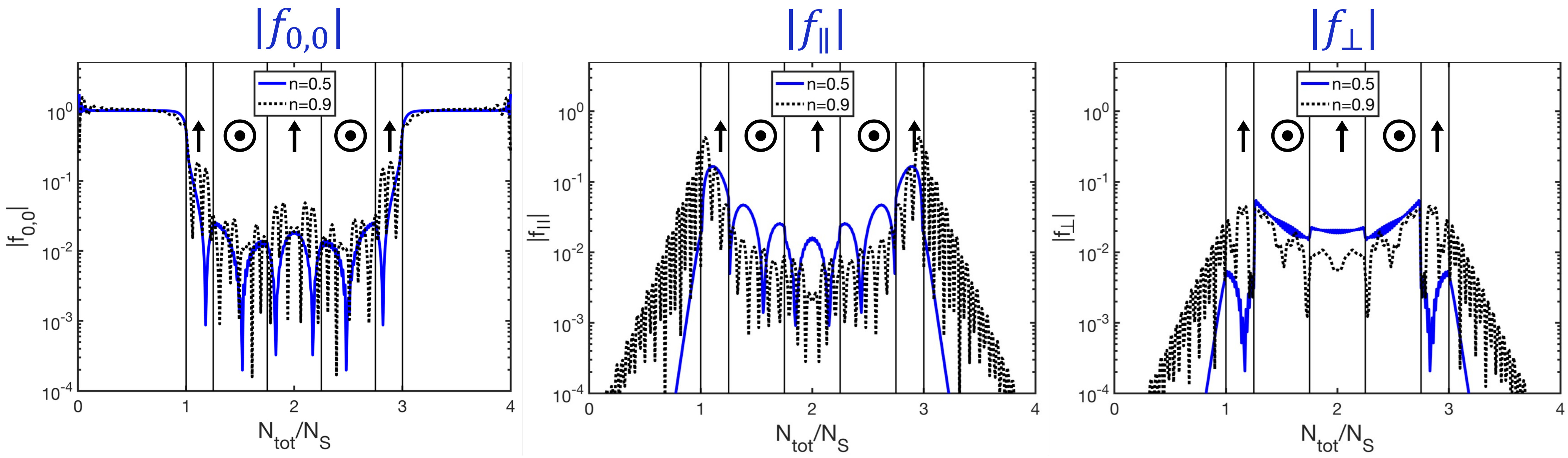


Filling: n = 0.9 (Nearly-full)



S5FS Junction: Band Filling Dependence

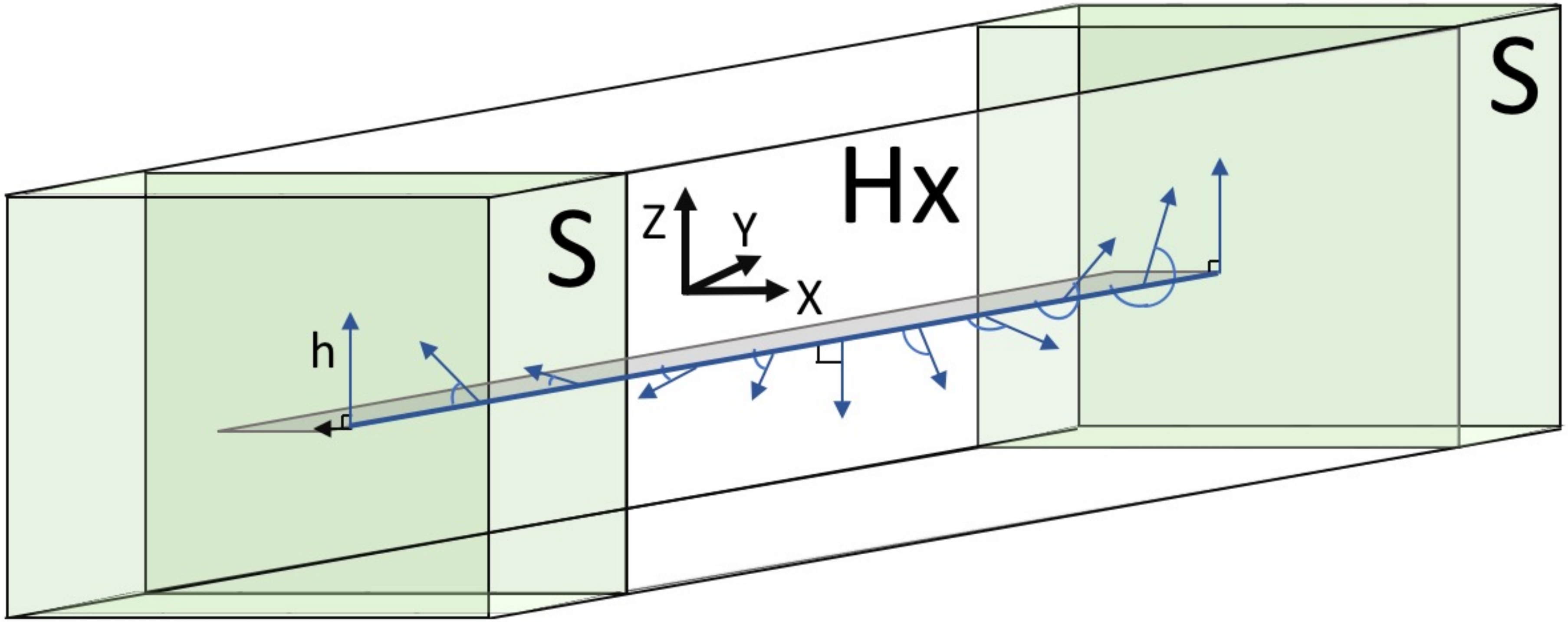
$n = 0.5$
$\omega = 0.1t$


$|f_{0,0}|$ $|f_{\parallel}|$ $|f_{\perp}|$

Blue smooth curve is $\mathrm{n} = 0.5$
Black dotted curve is $\mathrm{n} = 0.9$

Continuous layers: Helical configuration

$$|0{,}0\rangle \Rightarrow \alpha_{0,0}|0{,}0\rangle + \alpha_{1,0}|1{,}0\rangle + \alpha_{1,1}|1{,}1\rangle + \alpha_{1,-1}|1,-1\rangle$$

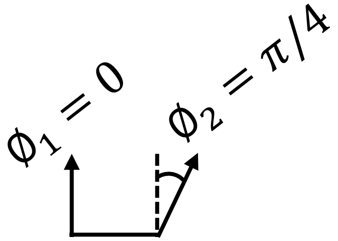


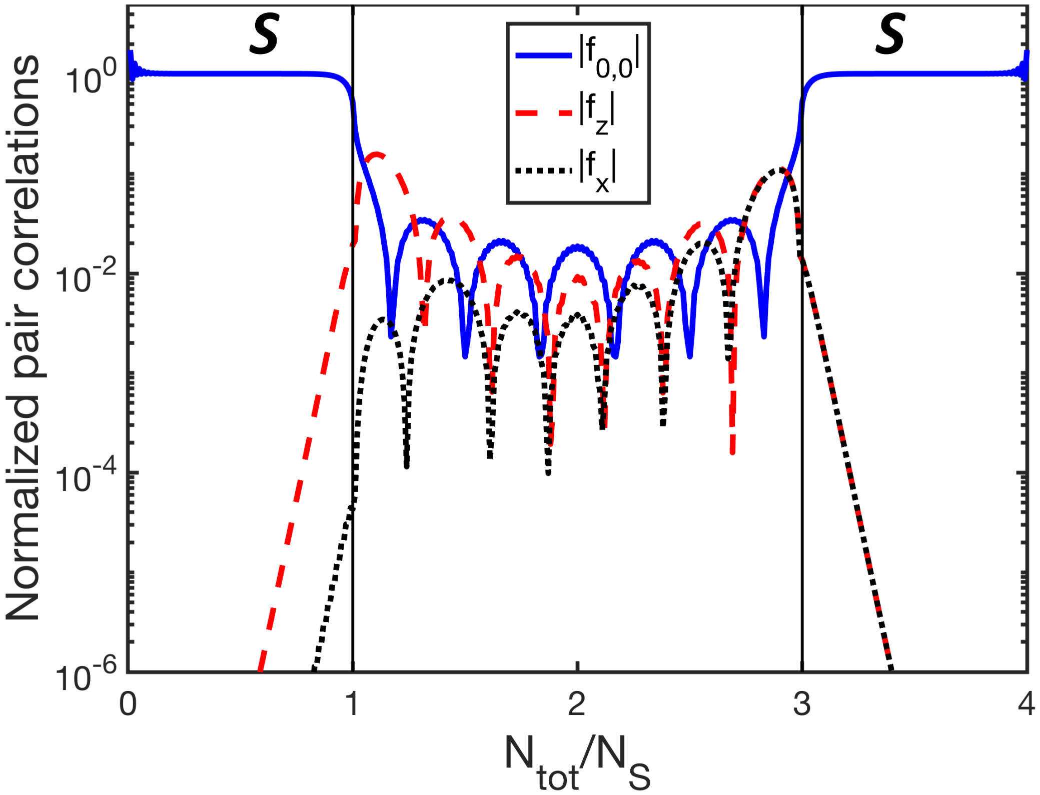
- Josephson junction with a continuous magnetic material.
- Helical configuration.

SHxS Junction: Rotating Basis

$n = 0.5$
$h = 0.1\text{t}$
$\omega = 0.1t$

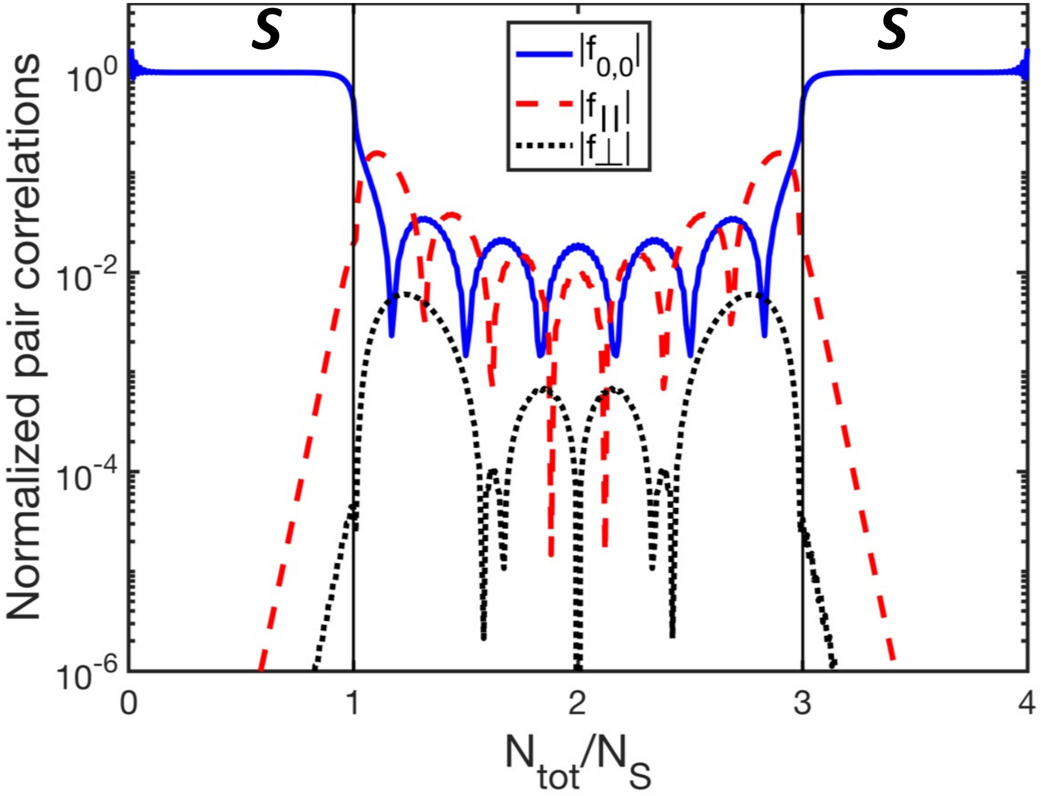
Rotation angle: $\Delta\emptyset = \frac{\pi}{4}$ →





Pair correlations in Cartesian (static) basis

$f_z \neq f_{1,0},\ f_x \neq f_{1,1}$



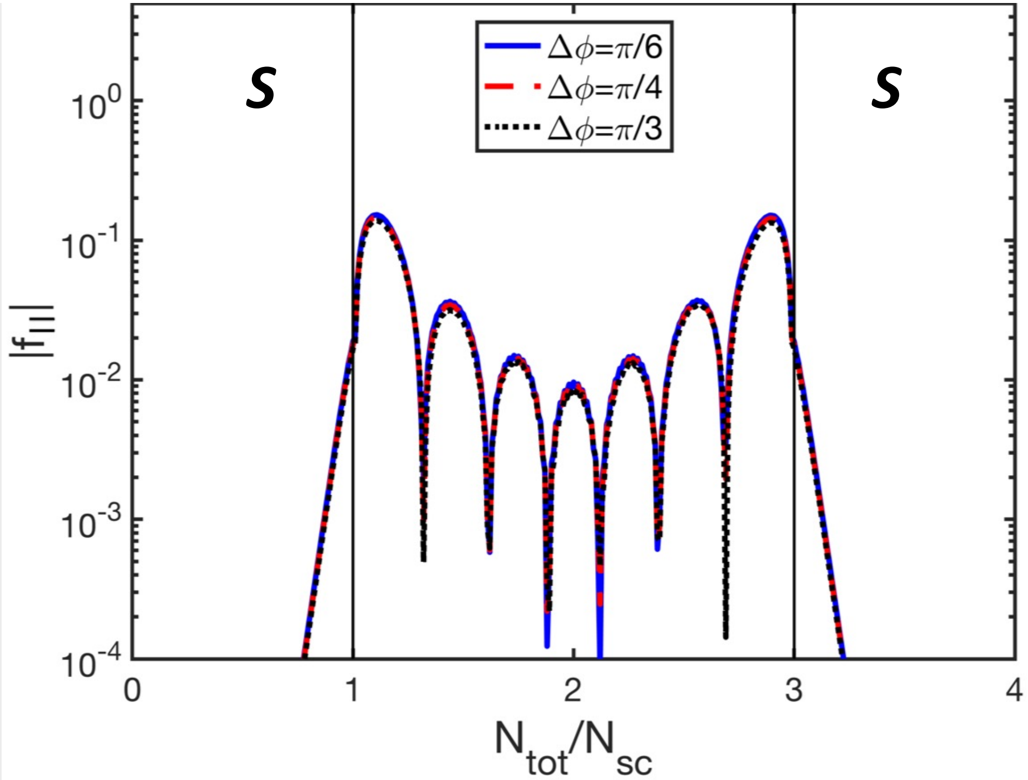
Pair correlations in rotating basis

$f_{\parallel} = f_{1,0},\ f_{\perp} = f_{1,1}$

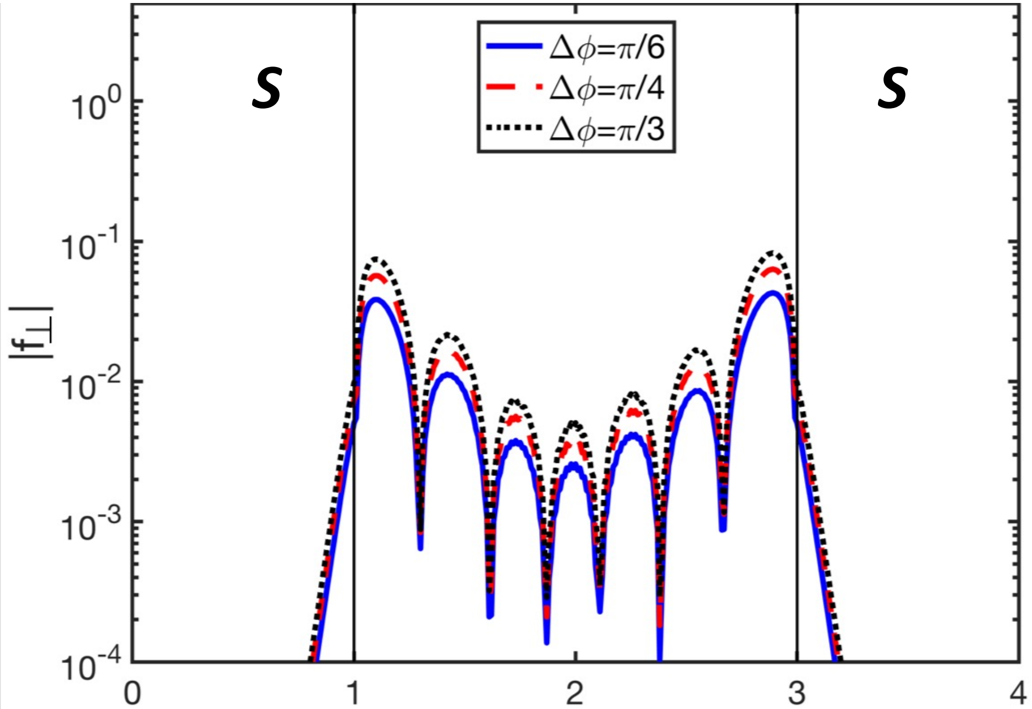
SHxS Junction: Rotation Angle Dependence

$n = 0.5$
$h = 0.1\text{t}$
$\omega = 0.1t$

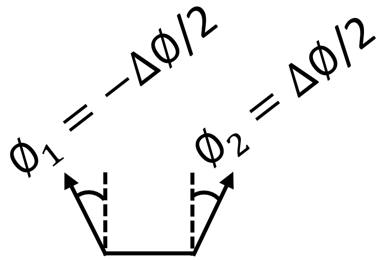
$|f_{\parallel}|$



$|f_{\perp}|$



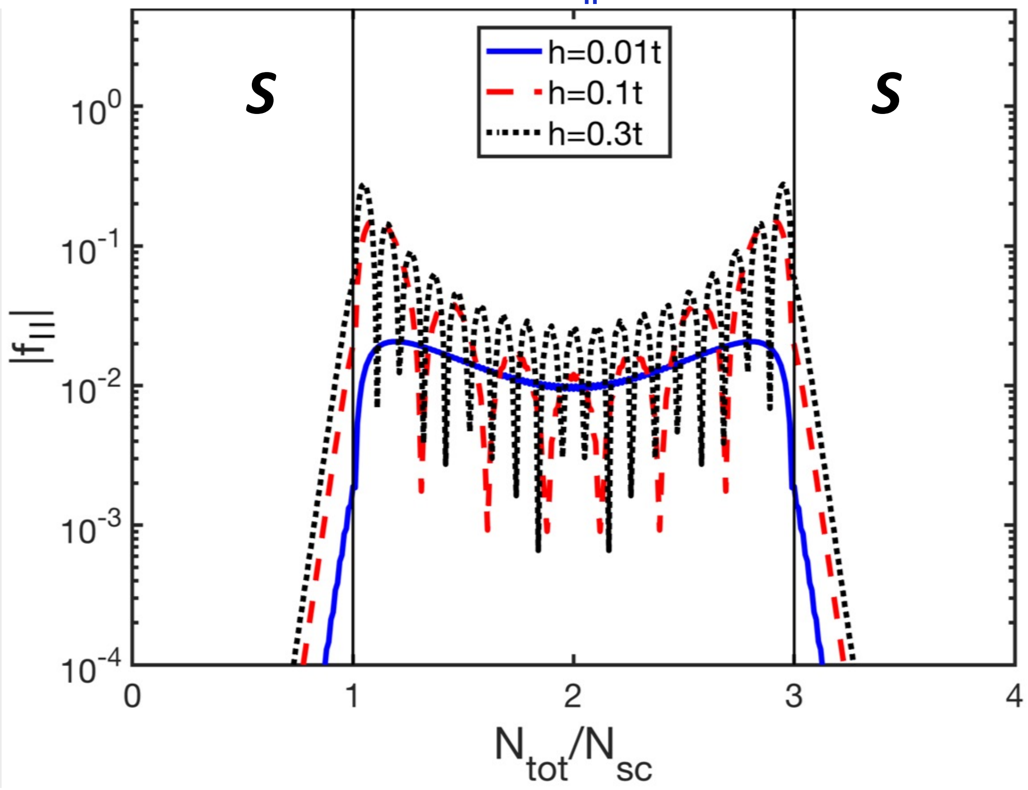
Rotation angle: $\Delta\emptyset$ ➡



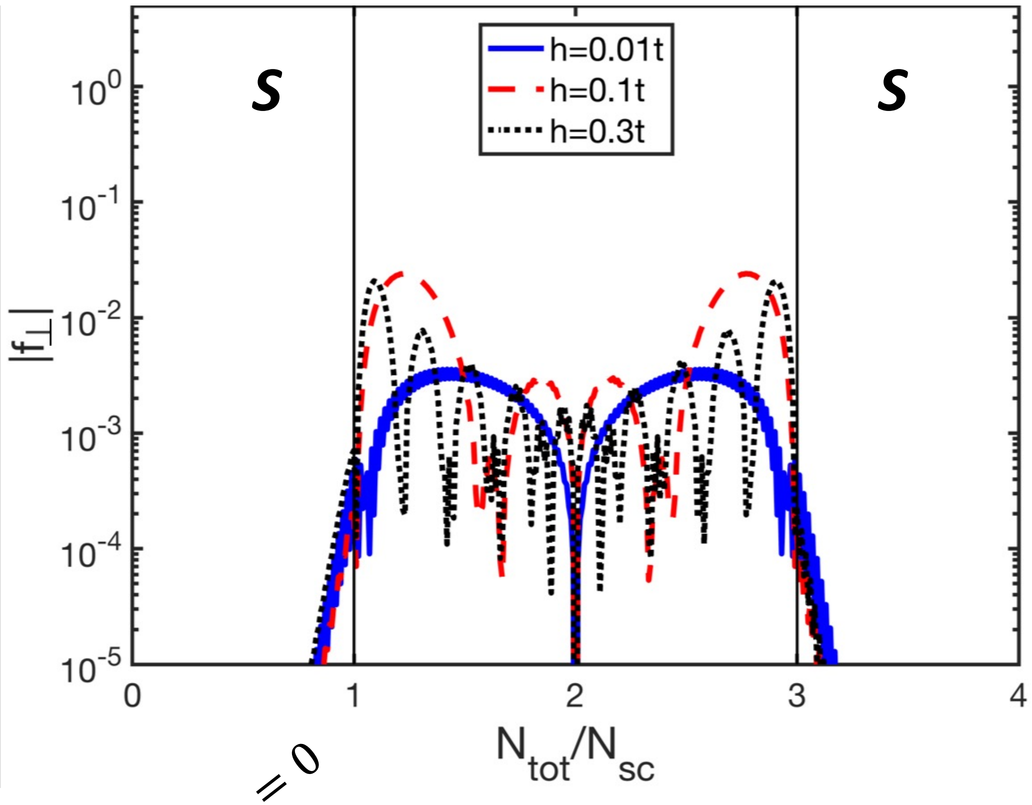
SHxS Junction: Magnetization Dependence

$n = 0.5$
$\omega = 0.1t$

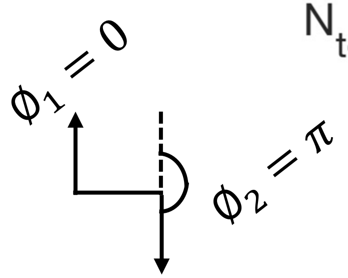
$|f_{\parallel}|$



$|f_{\perp}|$



Rotation angle: $\Delta\emptyset = \pi$ ➡

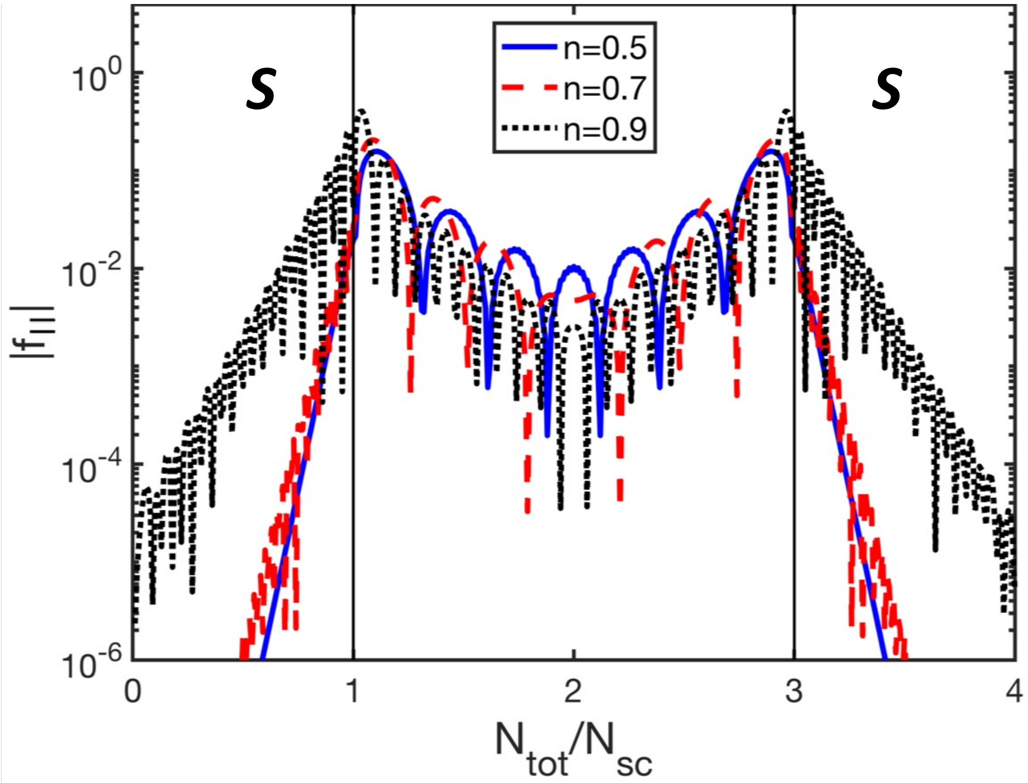


Bloch domain wall

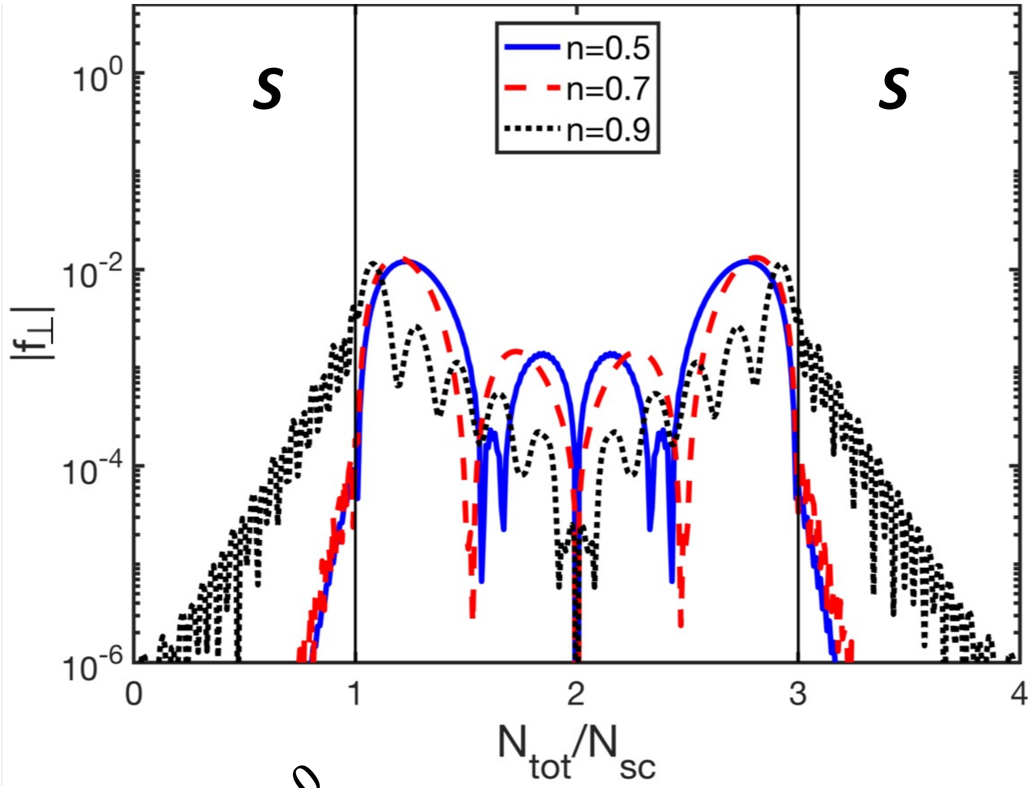
$h = 0.1\mathrm{t}$
$\omega = 0.1t$

SHxS Junction: Band Filling Dependence

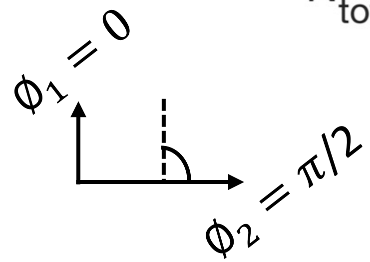
$|f_{\parallel}|$



$|f_{\perp}|$



Rotation angle: $\Delta\emptyset = \pi/2$ ➡



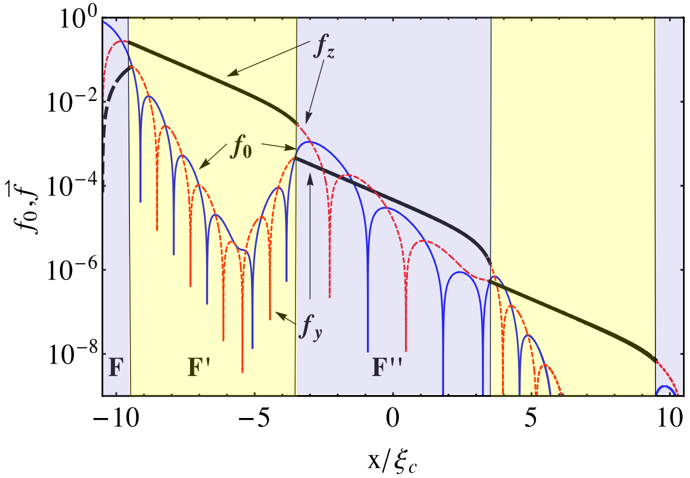
Ballistic vs. Diffusive regime: Comparison

- Diffusive regime: many nonmagnetic impurities.
- Clean regime: no impurities, strong dependence on chemical potential.

Diffusive regime

$$f_0(x) \propto e^{-x/\xi_F} \cos\left(\frac{x}{\xi_F}\right).$$

$$\xi_F = \sqrt{\frac{\hbar D_F}{2\pi h}}$$



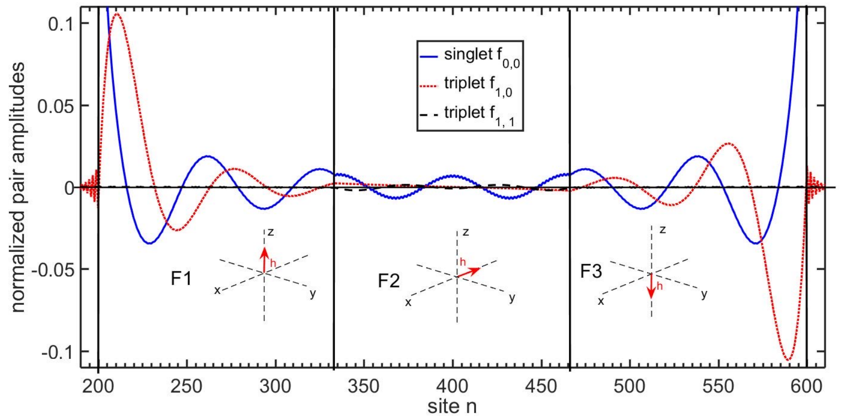
Half filling
(Log scale)

Clean regime

$$f_0(x) \propto \frac{1}{x} e^{-x/\xi_N} \cos\left(\frac{x}{\xi_F}\right)$$

$$\xi_N = \frac{v_F}{2\pi T}, \qquad \xi_F = \frac{v_F}{2h}$$

v_F depends on chemical potential

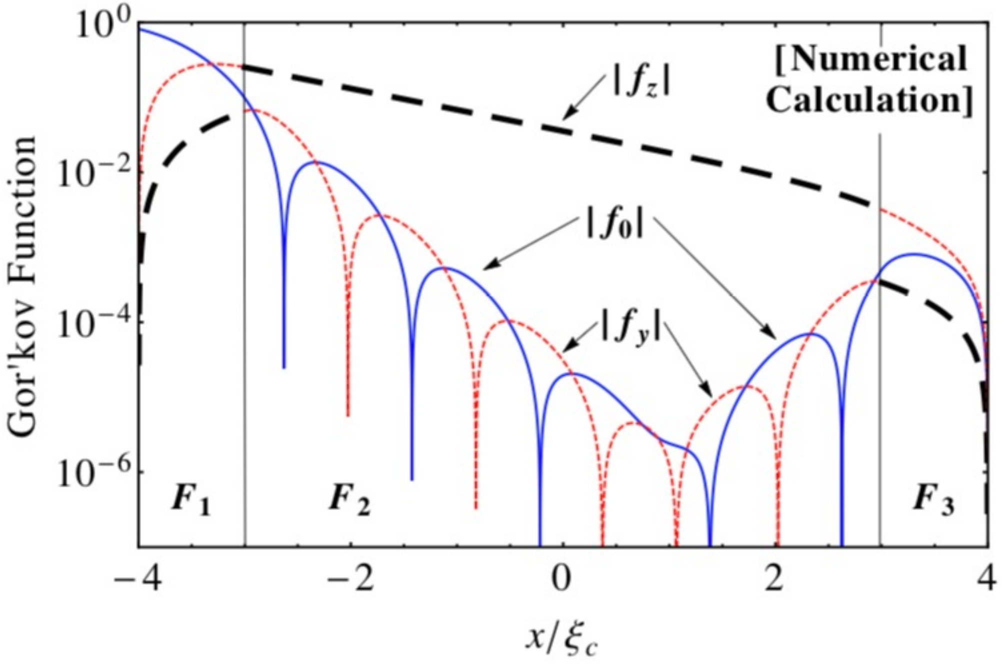


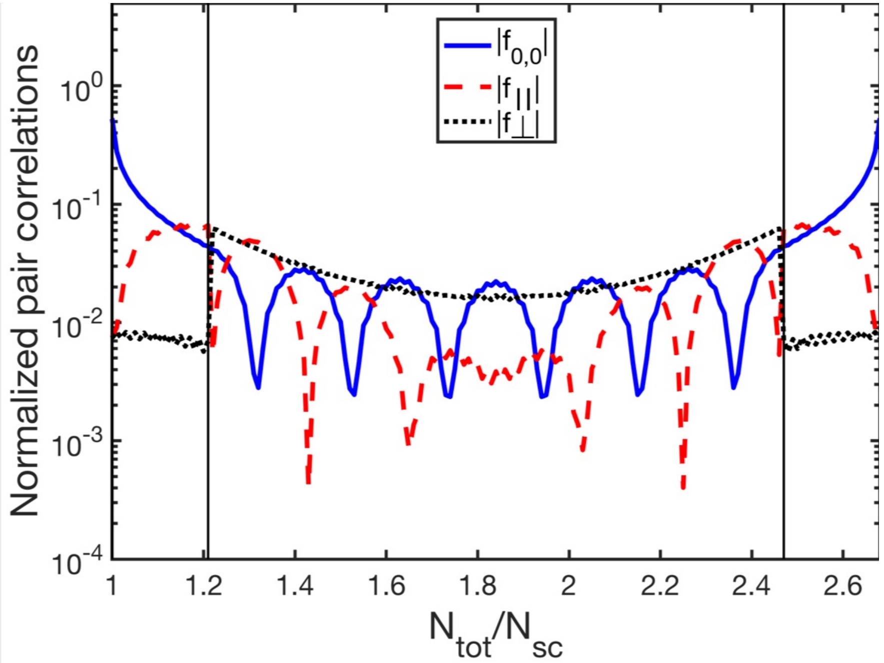
Half filling
(Linear scale)

$n = 0.5$
$\omega = 0.1t$
$T = 0.4T_c$

Ballistic vs. Diffusive Regime: S3FS Junction

- Magnetization is up, out, up





$h_i = (3, 14, 3)\pi T_c$

$h_i = (13, 59, 13)\pi T_c$

$N_{F_i} = (1, 6, 1)\xi_c$ → ξ_c = 21 sites

Conclusion:

- We studied how different magnetic configurations alter the superconducting state of the hybrid structure in clean limit.
- We observe that
  - Singlet pair correlations transform into a linear combination of all four basis states of spin ½ fermions pairs,
  - Pair correlations "bounce back" into the superconductor,
  - All pair correlations appear when the magnetization rotates.
  - A rotating basis disentangled the triplets.
  - Singlets in the clean limit behave different than in the dirty limit.

Future work:

- Determine the Josephson critical current.